

# Einstein Equations From Holographic Thermodynamics and Holographic Entropy

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**ABSTRACT:** We derive the Einstein field equations and black hole entropy from the first law of thermodynamics on a holographic time-like screen. Because of the universality of gravity, the stress tensor on the screen must be independent of the details of matter fields, so it should be a pure geometric quantity. For simplicity, we assume that the stress tensor on the screen depends on surface Ricci curvature and extrinsic curvature linearly. Then we prove that the surface stress tensor is just the Brown-York stress tensor plus terms which do not affect the field equations of gravitation and the entropy of the system. By assuming a generalized “Fine first law of thermodynamics” or the usual universal first law of thermodynamics on the screen, we can derive the matter field equations as well.

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## 1 Introduction

Ever since the discoveries of Hawking radiation [1] and Black hole entropy [2], the holographic viewpoint of gravity and the relationship between gravity and thermodynamics have draw much attention and brought about many research fields.

Along the line of holographic viewpoint of gravity, 't Hooft [3] and Susskind [4] proposed the holographic principle [5] for gravity theory, which states that the description of a volume of space can be thought of as encoded on a boundary to the region. The holographic principle was realized in an exact example known as AdS/CFT by Maldacena [6], who claims that the quantum gravity theory in  $AdS_5 \times S^5$  spacetime is dual to the  $N = 4$  Super-Yang-Mills theory without gravity on the boundary. It should be mentioned that before Maldacena, Brown and Henneaux had found that the asymptotic algebra of 2+1 D gravity in asymptotic  $AdS_3$  space is a Virasoro algebra of 2 D CFT, which can be regarded as the first example of *AdS/CFT* though the authors did not make such claims in their paper [7].

Along another line of the connection between gravity and thermodynamics, a noticeable breakthrough was made by Jacobson in 1995 [8], who first claims that gravity may not be a fundamental interaction but emerges from the first law of thermodynamics on a local Rindler horizon. This is a big claim on the origin of gravity, although Jacobson only succeeded in deriving the null-null component of Einstein equation. His work was then generalized to the case of higher derivative gravity [9, 10]. Recently, another significant breakthrough was made by Verlinde [11](see also Padmanabhan [12]), who states that the entropy will increase when particles pass through a holographic screen, which causes the emergence of gravity as an entropic force. Assuming the equipartition rule and the Tolman-Komar mass, Verlinde succeeded to obtain the time-time component of Einstein equation. There are many advantages in Verlinde's approach, in particular, the holographic screen can be placed at any place rather than only null surface in Jacobson's approach. However, as pointed out in [13], there is negative-temperature problem when one locates the closed

screen arbitrarily. There have appeared many following up papers [14] together with some criticism [15] after Verlinde's excellent work. However, it is still too early to say whether the thermodynamic viewpoint of gravity is right or not.

In this paper, we combine the spirits of holographic and thermodynamic viewpoints of gravity and prove that gravity can emerge from the first law of thermodynamics on a time-like screen. With the requirement that the entropy is a total differential, we succeed in deriving all the components of Einstein equations and proving that the entropy of a stationary black hole is the Bekenstein-Hawking entropy. In our approach, the key hypothesis is that the stress tensor on the screen depends on the surface Ricci curvature and extrinsic curvature linearly, which is quite reasonable and natural. Because gravity is universal, the surface stress tensor must be independent of the details of matter fields, so it should be purely geometrical. For simplicity, we assume that it depends on surface Ricci curvature and extrinsic curvature only linearly. Then, we prove that the stress tensor is just Brown-York stress tensor plus terms irrelevant with the bulk field equations and entropy of the system. It should be stressed that one may derive the higher derivative gravity with a more general assumption of the surface stress tensor. However, we only consider the simplest case in this paper. Finally, we want to mention that we can also derive Einstein equations from the entropy production of hydromechanics on the holographic screen [16].

The paper is arranged as follows. In Sec. 2, we give a brief review of thermodynamic viewpoint of gravity. In Sec. 3, we derive the vacuum Einstein equations from the first law of thermodynamics on a holographic time-like screen. In Sec. 4, we generalize our holographic scheme to the cases of the Einstein equations with matter fields. We conclude in Sec. 5.

## 2 Brief review of thermodynamic viewpoints of gravity

In this section, we shall briefly review the work of Verlinde [11] and Jacobson [8], and then compare their approaches with ours of the next section.

First, let us review Verlinde's approach of the derivations of the Einstein equation. According to [17], the Newton's potential is

$$\phi = \log(-\xi^\mu \xi_\mu), \quad (2.1)$$

where  $\xi^\mu$  is a local time-like Killing vector. Then we can write the Unruh temperature as

$$T = \frac{\hbar}{2\pi} e^\phi n^\mu \nabla_\mu \phi, \quad (2.2)$$

where  $n^\mu$  is a unit vector normal to the time-like surface  ${}^3B$  (please refer to the next section for the definition of  ${}^3B$ ).

The key assumptions of Verlinde's approach are the equipartition theorem and the Tolman-Komar mass

$$M = \frac{TN}{2} = \frac{TA}{2G\hbar}, \quad (2.3)$$

$$M = 2 \int_V (T_{\mu\nu} - \frac{T}{2} g_{\mu\nu}) u^\mu \xi^\nu dV, \quad (2.4)$$

where  $N = \frac{A}{G\hbar}$  is supposed to be the number of degrees of freedom on the screen,  $V$  is a spacelike hypersurface denoting the space volume and  $u^\mu$  is the unit vector normal to  $V$ . It should be mentioned that the equipartition rule of gravity eq.(2.3) was first derived by Padmanabhan [18]. Substituting eq.(2.2) into eq.(2.3), we derive [11]

$$M = \frac{TN}{2} = \frac{TA}{2G\hbar} = \frac{1}{4\pi G} \int_V R_{\mu\nu} u^\mu \xi^\nu dV. \quad (2.5)$$

Equating eq.(2.4) and eq.(2.5), we obtain the Einstein equations in an integral form

$$\int_V R_{\mu\nu} u^\mu \xi^\nu dV = 8\pi G \int_V (T_{\mu\nu} - \frac{T}{2} g_{\mu\nu}) u^\mu \xi^\nu dV. \quad (2.6)$$

Note that  $u^\mu, \xi^\nu$  are both time-like, so strictly speaking, we can only derive the time-time (tt) component of Einstein equations from eq.(2.6).

Now, let us turn to Jacobson's approach to derive the Einstein equations on a null surface. For simplicity, we use a slightly different method from Jacobson's initial approach. The key hypothesis of Jacobson's approach is that the entropy is proportional to area of the null surface

$$S = \eta A, \quad (2.7)$$

with  $\eta$  a constant. Jacobson also assumes that the first law of thermodynamics is satisfied on the null surface, thus we have

$$\delta Q = T \delta S, \quad (2.8)$$

with  $T$  the Unruh temperature. Note that on a null surface we have

$$T = \frac{\kappa}{2\pi} = \frac{1}{2\pi} k^\mu l^\nu \nabla_\mu \xi_\nu, \quad (2.9)$$

where  $\kappa$  is the surface gravity,  $k^\mu = \frac{dx^\mu}{d\lambda}$  is the tangent vector to the horizon,  $\lambda$  is the affine parameter,  $l^\nu$  is an auxiliary null vector with the property  $l^\mu k_\mu = -1$ . Using  $k^\mu \nabla_\mu k^\nu = 0$  and  $\xi^\mu = -\lambda \kappa k^\mu$  on horizon, we can prove eq.(2.9).

Substituting eqs.(2.7,2.9) into eq.(2.8), we can derive

$$\begin{aligned} \delta Q &= T \delta S = \eta \frac{\kappa}{2\pi} (dA|_\lambda^{\lambda+d\lambda}) \\ &= \frac{\eta}{2\pi} \int_{2H} (k^\mu l^\nu \nabla_\mu \xi_\nu dA)|_\lambda^{\lambda+d\lambda} \\ &= \frac{\eta}{2\pi} \int_H k^\mu \nabla^\nu \nabla_\mu \xi_\nu dA d\lambda \\ &= \frac{\eta}{2\pi} \int_H k^\mu R_{\mu\nu} \xi^\nu dA d\lambda. \end{aligned} \quad (2.10)$$

In the above derivations, we have used Stokes's Theorem and the identity  $\nabla_\nu \nabla_\mu \xi^\nu = R_{\nu\mu} \xi^\nu$ .

**Table 1.** comparisons between Verlinde's, Jacobson's and our methods

	assumption1	assumption2	Einstein equations
Verlinde	Equipartition rule $M = NT/2$	Tolman-Komar mass	tt component
Jacobson	The first law $\delta Q = T\delta S$	Entropy $S \sim A$	kk(null) component
ours	The first law $\delta S = \beta(\delta E + p\delta A - \omega^a\delta J_a)$	Surface stress tensor $\tau_{ij}$	All the components

According to [8], all the heat flux passing through the horizon is carried by matter

$$\delta Q = \int_H T_{\mu\nu} k^\mu \xi^\nu dA d\lambda. \quad (2.11)$$

In other words, the energy carried by matter becomes pure heat when it passes through the horizon. That is because the energy of matter inside the horizon is macroscopically unobservable. From eqs.(2.10,2.11) and  $k^\mu \xi_\mu = 0$ , we can derive

$$R_{\mu\nu} + f g_{\mu\nu} = \frac{2\pi}{\eta} T_{\mu\nu}. \quad (2.12)$$

From the conservation of energy of matter  $\nabla_\mu T^{\mu\nu} = 0$ , we get  $f = -\frac{R}{2} + \Lambda$ . Set  $\eta = \frac{1}{4G}$ , we obtain the Einstein equations

$$R_{\mu\nu} - \frac{R}{2} g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}. \quad (2.13)$$

Strictly speaking, we can only derive the null-null (kk) component of Einstein equations from eqs.(2.10,2.11).

To end this section, let us compare the approaches of Verlinde, Jacobson with ours of the next section in Table.1.

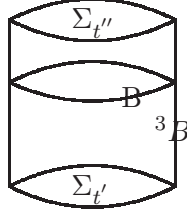
### 3 From holographic thermodynamics to vacuum Einstein equations

In this section, we shall derive the vacuum Einstein equations from the first law of thermodynamics on a time-like screen.

We use the notations of [19, 20] in this paper. Let us make a brief review of these notations.  $\Sigma_t$  is used to denote a family of spacelike slices that foliate  $M$ . The boundary of  $\Sigma_t$  is  $B$ , which is supposed to be closed. The product of  $B$  with segments of timelike world lines normal to  $\Sigma_t$  at  $B$  is denoted as  ${}^3B$ , the time-like three-boundary of  $M$ . The above notations are depicted in fig.1.

The metric and covariant derivative in  $M$  are denoted by  $g_{\mu\nu}$  and  $\nabla_\mu$ , respectively.  $n^\mu$  is the outward pointing unit vector normal to  ${}^3B$ .  $\gamma_{\mu\nu} = g_{\mu\nu} - n_\mu n_\nu$  is the metric on  ${}^3B$  with the corresponding covariant derivative  $D_\mu$ . The extrinsic curvature on  ${}^3B$  is defined as

$$\Theta_{\mu\nu} = -\gamma_\mu^\alpha \nabla_\alpha n_\nu. \quad (3.1)$$



**Figure 1.** Spacetime  $M$  with its three-boundary consisting of the spacelike hypersurface  $\Sigma_{t'}$ ,  $\Sigma_{t''}$  and timelike hypersurface  ${}^3B$ , the spacelike two-boundary of  $\Sigma_t$  is  $B$ .

Let  $x^i$  ( $i = 0, 1, 2$ ) be the intrinsic coordinates on  ${}^3B$ , then we can rewrite the metric, covariant derivative and the extrinsic curvature on  ${}^3B$  as  $\gamma_{ij}$ ,  $D_i$  and  $\Theta_{ij}$  with only intrinsic three indexes. Let  $u_\mu$  be the future pointing unit vector normal to  $\Sigma_t$ , then the metric and extrinsic curvature on  $\Sigma$  can be defined as  $h_{\mu\nu} = g_{\mu\nu} + u_\mu u_\nu$  and  $K_{\mu\nu} = -h_\mu^\alpha \nabla_\alpha u_\nu$ , respectively. For simplicity, we require that  $(u \cdot n)|_{{}^3B} = 0$  on the time-like screen as [19, 20], which means that the observer co-moving with  ${}^3B$  is static with respect to hypersurfaces  $\Sigma_t$ . With the above restriction, the metric on  ${}^3B$  can be decomposed as

$$\gamma_{ij} dx^i dx^j = -N^2 dt^2 + \sigma_{ab} (dx^a + V^a dt)(dx^b + V^b dt), \quad (3.2)$$

where  $x^a$  ( $a = 1, 2$ ) and  $\sigma_{ab}$  are coordinates and metric on  $B$ , respectively.

Let us begin to define various thermodynamic quantities on the time-like screen  ${}^3B$ . Suppose that the stress tensor on the time-like screen is  $\tau^{ij}$ . Now we do not assume any specific form of  $\tau^{ij}$ . In fact, one of the main purposes of this paper is to find a suitable expression for  $\tau^{ij}$ . Using the surface stress tensor  $\tau^{ij}$ , we can define the energy density  $\varepsilon$ , angular momentum density  $j_a$  and spatial stress  $s^{ab}$  on the time-like screen as follows:

$$\varepsilon = u_i u_j \tau^{ij}, \quad (3.3)$$

$$j_a = -\sigma_{ai} u_j \tau^{ij}, \quad (3.4)$$

$$s^{ab} = \sigma_i^a \sigma_j^b \tau^{ij}, \quad (3.5)$$

where  $u_i = (-N, 0, 0)$  is the speed of observer and  $\sigma_{ai}$  is the projection operator from  ${}^3B$  to space-like two-boundary  $B$  [19, 20]. As usual, we can define the energy  $E$ , angular momentum  $J_a$ , angular velocity  $\omega^a$  and pressure tensor  $p^{ab}$  as

$$E = \int_B d^2x \sqrt{\sigma} \varepsilon, \quad (3.6)$$

$$J_a = \int_B d^2x \sqrt{\sigma} j_a, \quad (3.7)$$

$$\omega^a = \frac{V^a}{N}, \quad p^{ab} = s^{ab}, \quad (3.8)$$

where the pressure  $p = \frac{1}{2}s^{ab}\sigma_{ab}$  if  $B$  is homogeneous and isotropic. The first law of thermodynamics on the time-like screen is

$$\delta S = \beta(\delta E - \omega^a \delta J_a + p \delta A), \quad (3.9)$$

where  $S$  is the entropy,  $\beta = \frac{1}{T}$  is the inverse temperature, and  $A = \int_B dx^2 \sqrt{\sigma}$  is the area of the screen. In general, since the pressure on  $B$  is a tensor, we can rewrite the first law in a more general form as

$$\delta S = \beta \int_B d^2x [\delta(\sqrt{\sigma}\varepsilon) - \frac{V^a}{N} \delta(\sqrt{\sigma}j_a) + \frac{\sqrt{\sigma}}{2} s^{ab} \delta\sigma_{ab}]. \quad (3.10)$$

Now we start to discuss the suitable definitions of  $\beta$  on a time-like screen. To get some insights for the definitions of  $\beta$ , let us first consider a special case of static black hole surrounded by a screen. Suppose that the inverse temperature of black hole is  $\beta_0$ , then the inverse temperature on the screen is  $\beta = N\beta_0$ , where  $N$  is the red-shift factor on the screen. Note that for a static black hole,  $\beta_0$  is the period of the Euclidean time  $\tau = it$ ,  $\beta_0 = \oint d\tau$ . So we get  $\beta = \oint d\tau N = i \int dt N$ . Based on the above discussions, we assume  $\beta$  can be defined as

$$\beta = i \int dt N|_B, \quad (3.11)$$

with  $t$  a pure imaginary number. For simplicity, we redefine  $(it)$  as  $t$ . Now  $t$  is a real number and the first law eq.(3.10) becomes

$$\delta S = \int dt \int_B d^2x N [\delta(\sqrt{\sigma}\varepsilon) - \frac{V^a}{N} \delta(\sqrt{\sigma}j_a) + \frac{\sqrt{\sigma}}{2} s^{ab} \delta\sigma_{ab}]. \quad (3.12)$$

Let us go on to discuss the suitable definition of stress tensor  $\tau^{ij}$  on a time-like screen. First, we aim to derive the field equations of gravitation in the bulk from the first law of thermodynamics on a screen. Due to the universality of gravity,  $\tau^{ij}$  must be independent of the details of matter fields. Thus,  $\tau^{ij}$  should be a pure geometric quantity. Second, we require  $\tau^{ij}$  to be a tensor on the screen so that the entropy  $S$  and gravitational field equations to be derived are independent of the choices of coordinates on the screen. Third, note that on a  $1+2$   $D$  screen, there are only three independent second-order tensors: the metric  $\gamma_{ij}$ , Ricci curvature  $\bar{R}_{ij}$  and extrinsic curvature  $\Theta_{ij}$ . For simplicity, we assume that  $\tau_{ij}$  is only linearly dependent of Ricci curvature and extrinsic curvature:

$$\tau_{ij} = c_1 \bar{R}_{ij} + c_2 \Theta_{ij} + f \gamma_{ij}, \quad (3.13)$$

where  $c_1, c_2$  are two constants and  $f$  is a function to be determined. It should be stressed that  $\tau_{ij}$  must depend on the extrinsic curvature  $\Theta_{ij}$ . Since the intrinsic geometry of the screen and the bulk geometry are independent, if  $\tau_{ij}$  depends only on the intrinsic geometry  $\gamma_{ij}$ ,  $\bar{R}_{ij}$  of the screen, we could not get any information of the dynamics of bulk geometry. On the other hand, the extrinsic curvature contains both the information of bulk and screen. Thus, it is a natural holographic bridge between the thermodynamics on the screen and the dynamics of geometry in the bulk.

Now with the suitable hypothesis of inverse temperature eq.(3.11) and surface stress tensor eq.(3.13), let us find out what can we obtain from the first law of thermodynamics eq.(3.12) on the holographic screen. Substituting eq.(3.13) into eq.(3.12), we can derive

$$\begin{aligned}
\delta S &= \int_{^3B} dt d^2x N [\delta(\sqrt{\sigma}\varepsilon) - \frac{V^a}{N} \delta(\sqrt{\sigma}j_a) + \frac{\sqrt{\sigma}}{2} s^{ab} \delta\sigma_{ab}] \\
&= \delta S_0 + \frac{c_2}{2} \int_M d^4x \sqrt{-g} (R^{\mu\nu} - \frac{R}{2} g^{\mu\nu} + \Lambda g^{\mu\nu}) \delta g_{\mu\nu} \\
&\quad - \frac{c_2}{2} \int_{\Sigma} \sqrt{h} d^3x (K h^{ij} - K^{ij}) \delta h_{ij}|_{t'}'' \\
&\quad - \int_{^3B} d^3x \sqrt{-\gamma} \delta(f + c_2 \Theta + \frac{c_1 \bar{R}}{2})
\end{aligned} \tag{3.14}$$

where  $S_0$  is

$$\begin{aligned}
S_0 &= \frac{c_2}{2} \int_M d^4x \sqrt{-g} (R - 2\Lambda) + c_2 \int_{\Sigma} \sqrt{h} d^3x K|_{t'}'' \\
&\quad - c_2 \int_{^3B} d^3x \sqrt{-\gamma} t_i \Theta^{ij} \partial_j t - c_1 \int_{^3B} d^3x \sqrt{-\gamma} t_i \bar{R}^{ij} \partial_j t
\end{aligned} \tag{3.15}$$

with  $t^i = Nu^i + V^j$  and  $\partial_i t = -u_i/N$ . Note that the terms on the space-like boundary  $\Sigma$  vanish when  $M$  has a topology  $M = (disk) \times S^2$  with  $\partial M = S^1 \times S^2$ . That is because the manifolds considered here have a single boundary  $\partial M = ^3B$ . In a more general case, we should consider the terms on  $\Sigma$ . However, for simplicity, we shall consider the terms on  $\Sigma$  only in this section and ignore them in the following sections.

Because the entropy  $S$  is a total differential, the second terms of the second line of eq.(3.14), the third and fourth lines of eq.(3.14) must vanish. So, we can derive

$$S = S_0, \tag{3.16}$$

$$R_{\mu\nu} - \frac{R}{2} g_{\mu\nu} + \Lambda g_{\mu\nu} = 0, \tag{3.17}$$

$$\delta h_{ij}|_{\Sigma_{t'}} = \delta h_{ij}|_{\Sigma_{t''}} = 0, \tag{3.18}$$

$$f = -c_1 \frac{\bar{R}}{2} - c_2 \Theta + c_3, \tag{3.19}$$

where  $c_3$  is a constant and eq.(3.18) is the boundary condition on  $\Sigma$ . Now from the first law of thermodynamics on the screen, we have derived the vacuum Einstein equations, the formula of entropy and the surface stress tensor

$$\tau_{ij} = c_2 (\Theta_{ij} - \Theta \gamma_{ij}) + c_1 (\bar{R}_{ij} - \frac{\bar{R}}{2} \gamma_{ij}) + c_3 \gamma_{ij}. \tag{3.20}$$

Notice that the first term of eq.(3.20) is just the Brown-York quasilocal stress tensor [19] if we set  $c_2 = \frac{1}{8\pi G}$ . From eq.(3.14), it is easy to observe that the last two terms of the surface stress tensor eq.(3.20) do not affect the bulk field equations. Below, we shall prove that the last two terms do not affect the entropy eq.(3.15) either. For simplicity, we focus on the stationary spacetime. We only need to prove that the last term of eq.(3.15)



vanishes for stationary spacetime. Substituting  $\partial_t t = -u_i/N$  and  $\sqrt{-\gamma} = N\sqrt{\sigma}$  into the last term of eq.(3.15), we can derive

$$\begin{aligned}
& -c_1 \int_{^3B} d^3x \sqrt{-\gamma} t_i \bar{R}^{ij} \partial_j t \\
& = c_1 \int dt \int_B d^2x \sqrt{\sigma} t_i \bar{R}^{ij} u_j \\
& = c_1 \int dt \int_B d^2x \sqrt{\sigma} u^j D^i D_{[j} t_{i]} \\
& = c_1 \int dt \int_{\partial B} dx \sqrt{\sigma} u^j m^i D_{[j} t_{i]} = 0.
\end{aligned} \tag{3.21}$$

In the above derivations, we have used the fact that  $t^i = (1, 0, 0)$  is a Killing vector in stationary spacetime. So we have  $D_{(i} t_{j)} = 0$  and  $\bar{R}_{ij} t^j = D_j D_i t^j$ . We also used the property that  $B$  has no boundary. Now we find that the last two terms of the surface stress tensor eq.(3.20) have no relations with the entropy and the gravitational field equations. In fact, they are related to the freedom to choose the zero point of the energy according to [19]. We want to mention that we can add extra terms  $\tau_{ij}^e$  to eq.(3.20) if we relax the constraint that  $\tau_{ij}$  depends on  $\bar{R}_{ij}$  linearly. A general form of such terms are as follows:

$$\tau_{ij}^e = L\gamma_{ij} - 2P_i^{lmn} \bar{R}_{jlmn} + 4D^n D^m P_{inmj}, \tag{3.22}$$

where  $P^{ijkl} = \frac{\partial L}{\partial \bar{R}_{ijkl}}$  with  $L$  is a general scalar function of  $\gamma_{ij}$ ,  $\bar{R}_{ijkl}$ . One can prove that such terms eq.(3.22) do not affect the bulk field equations and the entropy in stationary spacetime.

Now let us turn to study the entropy eq.(3.15). First, we want to mention that if  $(N, V^i, h_{ij})$  is a solution of Einstein equations with time  $t$ , then  $(-iN, -iV^i, h_{ij})$  is a solution of Einstein equations with imaginary time ( $i t$ ). And all the extensive quantities such as the entropy, energy, angular momentum and the area remain invariant under such transformations ( $t \rightarrow it$ ,  $N \rightarrow -iN$ ,  $V^i \rightarrow -iV^i$ ,  $h_{ij} \rightarrow h_{ij}$ ) [20]. So we can rewrite the entropy as  $S(t, N, V^j, h_{ij}) = S(it, -iN, -iV^j, h_{ij})$  which is consistent with eq.(4.9) of [20] in stationary spacetime. According to [20], we can rewrite eq.(3.15) as

$$\begin{aligned}
S_0 = & \int_M d^4x (P^{ij} \dot{h}_{ij} - NH - V^i H_i) \\
& + \int_{^3H} d^3x \sqrt{\sigma} [n^i (\partial_i N) / (8\pi G) + 2n_i V_j P^{ij} / \sqrt{h}] \\
& - c_1 \int_{^3B} d^3x \sqrt{-\gamma} t_i \bar{R}^{ij} \partial_j t,
\end{aligned} \tag{3.23}$$

where we have set  $c_2 = \frac{1}{8\pi G}$  and  $^3H$  is the horizon when there is a black hole contained in the screen.  $H = -\frac{\sqrt{h}}{8\pi G} G_{\mu\nu} u^\mu u^\nu$  and  $H_i = -\frac{\sqrt{h}}{8\pi G} G_{\mu\nu} u^\mu h^\nu_i$  are the Hamiltonian and momentum constraints, respectively. For simplicity, we focus on the stationary spacetime. Then, the first line and last line of the above equation vanish due to stationarity and bulk field equations. Notice that we have  $N = V^i = 0$ ,  $\int dt = \beta_0$  and  $\beta_0 n^i \partial_i N = 2\pi$  on the

horizon [20]. So we can derive the entropy as

$$\begin{aligned} S &= S_0 = \frac{1}{8\pi G} \int dt \int d^2x \sqrt{\sigma} n^i \partial_i N \\ &= \frac{1}{4} \int d^2x \sqrt{\sigma} = \frac{A_H}{4}, \end{aligned} \quad (3.24)$$

which is just the Bekenstein-Hawking entropy.

To summarize, we have derived the vacuum Einstein equations from the first law of thermodynamics on a holographic time-like screen. We also obtain the correct black hole entropy. With the hypothesis that  $\tau_{ij}$  depends on surface Ricci curvature and extrinsic curvature linearly, we can prove that  $\tau_{ij}$  is just the Brown-York stress tensor (set  $c_2 = \frac{1}{8\pi G}$ ) plus terms which do not affect the entropy (in stationary spacetime) and the field equations of gravitation. We want to mention that for a more general hypothesis of  $\tau_{ij}$  one may derive other gravity theories. However, we only investigate the simplest case in this paper.

To end this section, let us study a simple example to help us understand the above holographic derivations of Einstein equations. For simplicity, we focus on a spherically symmetric static spacetime with the metric

$$ds^2 = -N^2 dt^2 + h^2 dr^2 + r^2 d\Omega^2, \quad (3.25)$$

where  $N$  and  $h$  are functions of  $r$  and some parameters such as mass or entropy. And the screen is chosen to be the hypersurface at fixed  $r$ . For simplicity, we suppose that the stress tensor on the screen takes the form of eq.(3.20)

$$\tau_{ij} = \frac{1}{8\pi G} (\Theta_{ij} - \Theta \gamma_{ij}) + c_1 (\bar{R}_{ij} - \frac{\bar{R}}{2} \gamma_{ij}) + c_3 \gamma_{ij}, \quad (3.26)$$

where we have set  $c_2 = \frac{1}{8\pi G}$  for simplicity. From eq.(3.6), we get

$$E = -\frac{r}{h} - 4\pi r^2 c_3 + 4\pi c_1, \quad (3.27)$$

$$p = \frac{1}{8\pi} \left( \frac{N'}{Nh} + \frac{1}{rh} \right) + c_3. \quad (3.28)$$

We suppose that  $N$ ,  $h$  are functions of the area  $A = 4\pi r^2$  and the entropy  $S$  ( $A$  and  $S$  are two independent thermodynamic quantities on the screen), then the first law of thermodynamics on the screen becomes

$$\begin{aligned} dS &= \beta(dE + p dA) \\ &= \beta \left[ \frac{1}{8\pi h} \left( \frac{h'}{h} + \frac{N'}{N} \right) dA + \frac{r}{h^2} (\partial_S h) dS \right] \end{aligned} \quad (3.29)$$

Since  $S$  is a total differential, we have

$$\frac{h'}{h} + \frac{N'}{N} = 0, \quad (3.30)$$

$$\beta \frac{r}{h^2} (\partial_S h) = 1. \quad (3.31)$$

It is interesting to note that eqs.(3.29-3.31) are independent of  $c_1$  and  $c_3$ , which is consistent with our above conclusion that the last two terms of eq.(3.20) do not affect the entropy and bulk field equations. From eq.(3.30), one can easily get  $h = c/N$ . By redefining  $t$ , we can set  $c = 1$ . So we get

$$h = \frac{1}{N}. \quad (3.32)$$

Now we suppose there is a horizon at  $r_H$  inside the screen. Following the standard procedure, to avoid the conical singularity at the horizon, we can derive the temperature of the horizon as  $T_H(S) = \frac{NN'}{2\pi}|_{r=r_H}$ . Then the inverse temperature on the screen is

$$\beta = \frac{N}{T_H(S)} = N\left(\frac{2\pi}{NN'}|_{r=r_H}\right) \quad (3.33)$$

Substituting eq.(3.32) and eq.(3.33) into eq.(3.31), we get

$$\frac{\partial N^2}{\partial S} = -2\frac{T_H(S)}{r}, \quad (3.34)$$

from which we can derive  $N^2 = c - \frac{2f(S)}{r}$  with  $df = T_H dS$ . By redefining  $t$ , we can again set  $c = 1$ . Thus, we get

$$N^2 = 1 - \frac{2f(S)}{r}. \quad (3.35)$$

Applying  $T_H(S) = \frac{NN'}{2\pi}|_{r=r_H=2f(S)}$  together with  $df(S) = T_H dS$  and eq.(3.35), we can derive

$$T_H = \frac{1}{8\pi f(S)}, \quad S = \frac{1}{16\pi T_H^2} = \frac{A_H}{4}. \quad (3.36)$$

Now we have derived the correct spherically symmetric static solution of vacuum Einstein equations and black hole entropy:

$$N^2 = 1/h^2 = 1 - \frac{\sqrt{S/\pi}}{r}, \quad S = \frac{A_H}{4}. \quad (3.37)$$

#### 4 From holographic thermodynamics to Einstein equations with matter

In this section, we generalize our holographic program to the case with matter fields. We assume the ‘‘Fine first law of thermodynamics’’ on the screen, with which we mean that there is a corresponding ‘‘fine matter term’’ added to the first law for every special kind of matter field. By assuming the ‘‘Fine first law of thermodynamics’’, we can not only derive the Einstein equations with matter but also the matter field equations. So it is a very powerful holographic program. The key point of this holographic program is to search for a suitable form of the ‘‘fine matter terms’’ on the screen. We take scalar field and electromagnetic field as examples below.

Let us firstly study the case of scalar field  $\phi$ . We suppose the “Fine first law of thermodynamics” on the screen is

$$\delta S = \beta(\delta E - \omega^a \delta J_a + p \delta A + F_\phi \delta \phi), \quad (4.1)$$

where

$$F_\phi = - \int_B d^2 x \sqrt{\sigma} n^\mu \partial_\mu \phi. \quad (4.2)$$

According to [21], an extensive variable is a function of the phase space coordinates on  $B$  only. Thus,  $\phi$  is an extensive variable and the first law of thermodynamics eq.(4.1) includes only variation of extensive variables.  $F_\phi$  is designed so that we can derive the usual scalar field equation in the bulk. We assume the surface stress tensor take the form of eq.(3.13) as in the above section. Following a similar program, we can derive

$$\begin{aligned} \delta S &= \int_{3B} dt d^2 x N [\delta(\sqrt{\sigma} \varepsilon) - \frac{V^a}{N} \delta(\sqrt{\sigma} j_a) + \frac{\sqrt{\sigma}}{2} s^{ab} \delta \sigma_{ab} - \sqrt{\sigma} n^\mu \partial_\mu \phi \delta \phi] \\ &= \delta S_1 + \frac{c_2}{2} \int_M d^4 x \sqrt{-g} (R^{\mu\nu} - \frac{R}{2} g^{\mu\nu} + \Lambda g^{\mu\nu} - \frac{1}{c_2} T^{\mu\nu}) \delta g_{\mu\nu} \\ &\quad - \int_{3B} d^3 x \sqrt{-\gamma} \delta(f + c_2 \Theta + \frac{c_1 \bar{R}}{2}) - \int_M d^4 x \sqrt{-g} [\square \phi - \frac{\partial V(\phi)}{\partial \phi}] \delta \phi, \end{aligned} \quad (4.3)$$

where  $S_1$  and  $T^{\mu\nu}$  are

$$S_1 = S_0 + \int_M d^4 x \sqrt{-g} [-\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi)], \quad (4.4)$$

$$T^{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta(S_1 - S_0)}{\delta g_{\mu\nu}}. \quad (4.5)$$

Since the entropy is a total differential, from eq.(4.3) we can derive the Einstein equations, scalar field equation, the entropy and the surface stress tensor as follows:

$$R_{\mu\nu} - \frac{R}{2} g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}, \quad (4.6)$$

$$\square \phi - \frac{\partial V(\phi)}{\partial \phi} = 0, \quad S = S_1, \quad (4.7)$$

$$\tau_{ij} = \frac{1}{8\pi G} (\Theta_{ij} - \Theta \gamma_{ij}) + c_1 (\bar{R}_{ij} - \frac{\bar{R}}{2} \gamma_{ij}) + c_3 \gamma_{ij}, \quad (4.8)$$

where we have set  $c_2 = \frac{1}{8\pi G}$ . Following the same method of Sec. 3, one can easily prove that the entropy of stationary black hole with scalar field is also  $S = \frac{A_H}{4}$ .

$$\begin{aligned} S &= \int_M d^4 x (P^{ij} \dot{h}_{ij} + P_\phi \dot{\phi} - NH - V^i H_i) \\ &\quad + \int_{3H} d^3 x \sqrt{\sigma} n^i (\partial_i N) / (8\pi G) \end{aligned} \quad (4.9)$$

where  $H = -\frac{\sqrt{h}}{8\pi G}(G_{\mu\nu} - 8\pi GT_{\mu\nu})u^\mu u^\nu$  and  $H_i = -\frac{\sqrt{h}}{8\pi G}(G_{\mu\nu} - 8\pi GT_{\mu\nu})u^\mu h^\nu_i$  are the Hamiltonian and momentum constraints. Note that, in general, we have  $\sqrt{-g}n^\mu\partial_\mu\phi \rightarrow 0$  as  $r \rightarrow \infty$ . Thus, the first law of thermodynamics eq.(4.1) reduces to the usual form  $\delta S = \beta(\delta E - \omega^a\delta J_a + p\delta A)$  on a screen placed at infinity. And the “fine matter term”  $F_\phi\delta\phi$  only becomes important on the screen at finite radius.

Finally, we want to clarify the confusion readers may have in our above derivations. It seems that there are arbitrary terms of potential energy  $V(\phi)$  and cosmological constant  $\Lambda$  in our above derivations. Thus, the same first law of thermodynamics on the screen appears to correspond to many different bulk field equations with different  $V(\phi)$  and  $\Lambda$ . This is however not the case. Notice that the entropy eq.(4.4) in the first law is different for different choice of  $V(\phi)$  and  $\Lambda$ . So one specific first law of thermodynamics on the screen corresponds to the unique bulk field equations. In fact, similar to the integral constant,  $\sqrt{-g}V(\phi)$  and  $\sqrt{-g}\Lambda$  can be regarded as the “integral constant functions” which do not affect the variational forms of the first law.

Now we turn to discuss the case of electromagnetic field. We develop a two-step program. In the first step, we focus on the derivations of bulk field equations, so we do not need all the details of the first law of thermodynamics on the screen. Instead, we leave a total differential surface term  $S_A$  free which does not affect the bulk field equations. In the second step, we choose  $S_A$  carefully so that the first law of thermodynamics on the screen includes only variation of extensive variables. And as a result, we can derive the entropy of the system.

First, we suppose the “Fine first law of thermodynamics” on the screen is

$$\delta S = \beta(\delta E - \omega^a\delta J_a + p\delta A + F_A^\mu\delta A_\mu) + \delta S_A, \quad (4.10)$$

where  $S_A$  is a total differential term which do not affect the bulk field equations and  $F_A^\mu$  is designed as

$$F_A^\mu = - \int_B d^2x \sqrt{\sigma} n_\nu F^{\nu\mu}, \quad (4.11)$$

in order to derive the Maxwell’s equations in the bulk. We assume the surface stress tensor eq.(3.13) as before. Following the same program, we can derive

$$\begin{aligned} \delta S &= \int_{3B} dt d^2x N [\delta(\sqrt{\sigma}\varepsilon) - \frac{V^a}{N}\delta(\sqrt{\sigma}j_a) + \frac{\sqrt{\sigma}}{2}s^{ab}\delta\sigma_{ab} - \sqrt{\sigma}n_\nu F^{\nu\mu}\delta A_\mu] + \delta S_A \\ &= \delta S_2 + \frac{c_2}{2} \int_M d^4x \sqrt{-g} (R^{\mu\nu} - \frac{R}{2}g^{\mu\nu} + \Lambda g^{\mu\nu} - \frac{1}{c_2}T^{\mu\nu})\delta g_{\mu\nu} \\ &\quad - \int_{3B} d^3x \sqrt{-\gamma} \delta(f + c_2\Theta + \frac{c_1\bar{R}}{2}) - \int_M d^4x \sqrt{-g} (\nabla_\mu F^{\mu\nu})\delta A_\nu, \end{aligned} \quad (4.12)$$

where  $S_2$  and  $T^{\mu\nu}$  are

$$\begin{aligned} S_2 &= S_0 + S_A - \frac{1}{4} \int_M d^4x \sqrt{-g} F_{\mu\nu} F^{\mu\nu}, \\ T^{\mu\nu} &= \frac{2}{\sqrt{-g}} \frac{\delta(S_2 - S_0 - S_A)}{\delta g_{\mu\nu}}. \end{aligned} \quad (4.13)$$

Because the entropy is a total differential, from eq.(4.12) we can derive the Einstein equations, Maxwell's equations and the surface stress tensor as follows:

$$\begin{aligned} R_{\mu\nu} - \frac{R}{2}g_{\mu\nu} + \Lambda g_{\mu\nu} &= 8\pi G T_{\mu\nu}, \quad \nabla_\mu F^{\mu\nu} = 0, \\ \tau_{ij} &= \frac{1}{8\pi G}(\Theta_{ij} - \Theta\gamma_{ij}) + c_1(\bar{R}_{ij} - \frac{\bar{R}}{2}\gamma_{ij}) + c_3\gamma_{ij}, \end{aligned} \quad (4.14)$$

where we have set  $c_2 = \frac{1}{8\pi G}$  as before.

Second, let us now study  $S_A$  carefully so that we can express the first law eq.(4.10) with only variation of the extensive variables obviously. Let us firstly rewrite the electromagnetic terms of eq.(4.10) as follows

$$\begin{aligned} \delta S_A &- \int_{3B} d^3x N \sqrt{\sigma} n_\nu F^{\nu i} \delta A_i, \\ &= \delta S_A + \int_{3B} d^3x \sqrt{\sigma} \{ n_\nu F^{\nu i} u_i [\delta(N u^j A_j) + \hat{A}_a \delta V^a] - N n_\nu F^{\nu a} \delta \hat{A}_a \}, \\ &= \delta S_A + \int_{3B} d^3x \{ -\tilde{Q} \delta(N\Phi) + \tilde{J}_a \delta V^a + N F^a \delta \hat{A}_a \}, \end{aligned} \quad (4.15)$$

where we have defined the electric potential  $\Phi = -u^j A_j$ , charge density  $\tilde{Q} = \sqrt{\sigma} n_\mu F^{\mu\nu} u_\nu$  and electric momentum density  $\tilde{J}_a = \tilde{Q} \hat{A}_a$  with  $\hat{A}_a = \sigma_a^i A_i$  and  $F^a = -\sqrt{\sigma} n_\nu F^{\nu a}$ . In the above derivations, we have also used the formula  $\delta A_i = N^{-1} u_i [\delta(N\Phi) - \hat{A}_a \delta V^a] + \sigma_i^a \delta \hat{A}_a$ . Since the extensive variables are functions of only the phase space coordinates on  $B$  [21],  $\tilde{J}_a$ ,  $\tilde{Q}$ ,  $\hat{A}_a$ ,  $\sigma_{ab}$  are all the extensive variables. Conversely,  $N$ ,  $V^a$ ,  $\Phi$  are the intensive variables. Now it is clear by choosing  $S_A$  as

$$S_A = \int_{3B} d^3x \{ \tilde{Q} N \Phi - \tilde{J}_a V^a \}, \quad (4.16)$$

the the first law of thermodynamics contains only variation of extensive variables:

$$\begin{aligned} \delta S &= \beta [\delta E - \omega^a \delta(J_a + \tilde{J}_a) + p \delta A + \Phi \delta Q + F^a \delta \hat{A}_a] \\ &= \int_{3B} dt d^2x N [\delta(\sqrt{\sigma} \varepsilon) - \frac{V^a}{N} \delta(\sqrt{\sigma} (j_a + \tilde{j}_a)) + \frac{\sqrt{\sigma}}{2} s^{ab} \delta \sigma_{ab} + \Phi \delta \tilde{Q} - \sqrt{\sigma} n_\nu F^{\nu a} \delta \hat{A}_a]. \end{aligned} \quad (4.17)$$

Now we get the expression of the entropy

$$S = S_0 - \frac{1}{4} \int_M d^4x \sqrt{-g} F_{\mu\nu} F^{\mu\nu} + \int_{3B} d^3x \{ \tilde{Q} N \Phi - \tilde{J}_a V^a \}. \quad (4.18)$$

Following the same method, one can prove  $S = \frac{A_H}{4}$  for stationary charged black holes. It is interesting that for Reissner-Nordstrom black hole, the first law reduces to the usual form

$$\delta S = \beta [\delta E + p \delta A + \Phi \delta Q]$$

on a time-like screen.

Now we have derived the Einstein equations, scalar field equation as well as Maxwell's equations from the “Fine first law of thermodynamics” on a holographic screen. One can obtain the field equations of other matter in a similar program. However, if we do not care about the details of matter fields, we can simplify the above derivations and develop a more universal holographic program. Consider a uncharged system surrounded by a screen at  $r \rightarrow \infty$ , all the matter fields vanish quickly enough as  $r$  approaches infinity so that the “Fine matter terms” become less important and in the leading order the first law of thermodynamics takes the universal form

$$\delta S = \beta(\delta E - \omega^a \delta J_a + p \delta A) + \dots \quad (4.19)$$

where ... means next leading order. Suppose the surface stress tensor eq.(3.13) as before, we can derive

$$\begin{aligned} \delta S &= \int_{3B} dt d^2 x N [\delta(\sqrt{\sigma} \varepsilon) - \frac{V^a}{N} \delta(\sqrt{\sigma} j_a) + \frac{\sqrt{\sigma}}{2} s^{ab} \delta \sigma_{ab}] + \dots \\ &= \delta S_3 + \frac{c_2}{2} \int_M d^4 x \sqrt{-g} (R^{\mu\nu} - \frac{R}{2} g^{\mu\nu} + \Lambda g^{\mu\nu} - \frac{1}{c_2} T^{\mu\nu}) \delta g_{\mu\nu} \\ &\quad - \int_{3B} d^3 x \sqrt{-\gamma} \delta (f + c_2 \Theta + \frac{c_1 \bar{R}}{2}) - \frac{\delta S_M(g_{\mu\nu}, \Psi)}{\delta \Psi} \delta \Psi \dots \end{aligned} \quad (4.20)$$

where  $S_3 = S_0 + S_M$  with  $S_M$  is the action of matter fields  $\Psi$ . From the above equation, we can derive the Einstein equations, matter field equations, the entropy in the leading order and the stress tensor:

$$\begin{aligned} R_{\mu\nu} - \frac{R}{2} g_{\mu\nu} + \Lambda g_{\mu\nu} &= 8\pi G T_{\mu\nu}, \\ \frac{\delta S_M}{\delta \Psi} &= 0, \quad S = S_0 + S_M + \dots \\ \tau_{ij} &= \frac{1}{8\pi G} (\Theta_{ij} - \Theta \gamma_{ij}) + c_1 (\bar{R}_{ij} - \frac{\bar{R}}{2} \gamma_{ij}) + c_3 \gamma_{ij}. \end{aligned} \quad (4.21)$$

## 5 Conclusions

In this paper, we have derived the Einstein equations and the black hole entropy from the first law of thermodynamics on a holographic time-like screen. Based on the reasonable hypothesis that the surface stress tensor depends on extrinsic curvature and surface Ricci curvature linearly, we prove that the stress tensor on the screen is just the Brown-York stress tensor plus terms which do not affect the gravitational field equation and the entropy of the system. Applying a generalized “fine first law of thermodynamics” or the usual first law of thermodynamics on the holographic screen, we can also derive the matter field equations. It is interesting to generalize our holographic approach to the case of higher derivative gravity. For example, if we suppose that the surface stress tensor takes a more general form

$$\tau_{ij} = F'(R) \Theta_{ij} + f \gamma_{ij} + c_1 \bar{R}_{ij}, \quad (5.1)$$

where  $R$  is the Ricci scalar in the bulk and  $F'(R) = \partial F(R)/\partial R$ , one may obtain the  $F(R)$  gravity from the first law of thermodynamics on the screen (one may need a extra ‘fine matter term’ for  $\phi = F'(R)$  in view of the equivalence between the scalar-tensor theory and  $F(R)$  gravity). Besides, it is also interesting to investigate our holographic approach in quantum level. For example, how to calculate the quantum corrections for black hole entropy, how to establish the quantum theory of matter fields and even the gravity in our holographic thermodynamics approach... We hope to address these issues in the following works.

To end this paper, we want to stress that, in certain sense, our holographic thermodynamics approach is equivalent to the action principle: they can be used to derive the same bulk field equations. However, they have several significant differences. First, they have different physical origins: our holographic approach is based on the first law of thermodynamics, or in other words, the conservation of energy on the screen; while the action principle is based on the principle of least action that the path taken by a particle is the one with least (extreme) action. Second, the action principle needs suitable boundary conditions in order to have a good definition of the variations of action. On the contrary, our holographic approach does not need such boundary conditions (at least on  ${}^3B$ ). Last but not least, our holographic approach depends on heavily the existence of gravity. One can not write a non-degenerate first law of thermodynamics on the screen if there is no gravitational interaction in the bulk. While there is no such constraint for the action principle: one can easily write a action of matter field without gravity. It seems that the action principle is more universal than our approach. This is however not the case. Taking into account the fact that all of the matter fields have gravitational interaction, one has to always consider the matter and gravity interaction together for every physical process. So the third point above is an advantage rather than weakness of our holographic approach: we predict the existence of gravity in certain sense just as the string theory predicts supersymmetry and the extra dimensions.

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## References

- [1] S. W. Hawking, *Nature* **248**, 30 (1974).
- [2] J. D. Bekenstein, *Phys. Rev. D* **7**, 2333 (1973).
- [3] G. 't Hooft, arXiv:gr-qc/9310026.
- [4] L. Susskind, *J. Math. Phys.* **36**(11), 6377 (1995).
- [5] R. Bousso, *Rev. Mod. Phys.* **74**, 825 (2002); W. Fischler, L. Susskind, arXiv:hep-th/9806039.



- [6] J. Maldacena, Adv. Theor. Math. Phys **2**, 231 (1998).
- [7] J.D. Brown and M. Henneaux, Commun. Math. Phys. **104**, 207 (1986).
- [8] T. Jacobson, Phys. Rev. Lett. **75**, 1260 (1995).
- [9] C. Eling, R. Guedens and T. Jacobson, Phys. Rev. Lett. **96**, 121301 (2006).
- [10] R. Brustein and M. Hadad, Phys. Rev. Lett. **103**, 101301 (2009).
- [11] E. P. Verlinde, JHEP 1104, 029 (2011).
- [12] T. Padmanabhan, Rept. Prog. Phys. **73**, 046901 (2010); Mod. Phys. Lett. A **25**, 1129 (2010).
- [13] W. Gu, M. Li and R. X. Miao, Sci. China G **54** 1915 (2011)[arXiv:1011.3419[hep-th]]; R. X. Miao, J. Meng and M. Li, Sci. China G **55** 375 (2012)[arXiv:1102.1166[hep-th]].
- [14] T. Padmanabhan, J. Phys 306, 1(2011), 12001-12018; T. Padmanabhan, arXiv: 1206.4916 [hep-th]; Yu Tian, Xiao-Ning Wu, JHEP 2011, 1(2011), 150; Kourosh Nozari, Siamak Akhshabi, Phy.Lett.B, 700, 2(2011), 91-96; R. B. Mann, J. R. Mureika, Phy. Lett. B, 703, 2(2011), 167-171; Taotao Qiu, Emmanuel N. Saridakis, Phys. Rev. D 85, 043504 (2012); A. Sheykhi, H. Moradpour and N. Riazi, arXiv:1109.3631v3 [physics.gen-ph]; A. Sheykhi, Z. Teimoori, Gen. Relativ. Gravit, 44, 5(2012), 1129-1141; Shu Zhu, Shao-Feng Wu, Guo-Hong Yang, Mod. Phys. Lett. A, 26,14(2011), 1025-1034; A. Sheykhi and K. R. Sarab, arXiv:1206.1030 [physics.gen-ph]; W. J. Jiang, Y. X. Chen and J. L. Li, arXiv:1206.3492 [hep-th]; R. Banerjee and B. R. Majhi, Phys. Rev. D **81**, 124006 (2010); M. Li and Y. Wang, Phys. Lett. B **687**, 243 (2010); R. G. Cai, L. M. Cao, N. Ohta, Phys. Rev. D **81**, 061501 (2010); W. Shu and Y. Gong, Int.J.Mod.Phys. D **20**, 553 (2011); C. Gao, Phys. Rev. D **81**, 087306 (2010); Y. X. Liu, Y. Q. Wang and S. W. Wei, Class. Quant. Grav. **27**, 185002 (2010); P. Nicolini, Phys. Rev. D **82**, 044030 (2010); T. Wang, Phys. Rev. D **81**, 104045 (2010). S. H. Mehdipour, A. Keshavarz, Europhys. Lett. 91, 10002 (2012). S. H. Mehdipour, arXiv:1111.2468 [gr-qc].
- [15] A. Kobakhidze, Phys. Rev. D **83**, 021502 (2011); M. Visser, JHEP **1110**, 140 (2011).
- [16] M. Li and R. X. Miao, in preparation.
- [17] R. M. Wald, University of Chicargo Press, 1994.
- [18] T. Padmanabhan, Class. Quan. Grav. **21**, 4485 (2004).
- [19] J. D. Brown and J. W. York, Phys. Rev. D **47**, 1407 (1993).
- [20] J. D. Brown and J. W. York, Phys. Rev. D **47**, 1420 (1993).
- [21] J. Creighton and R. B. Mann, Phys. Rev. D **52**, 4569 (1995).